# Applying Mathematical Knowledge in a Design-Based Interdisciplinary Project


Kit Ee Dawn Ng
*National Institute of Education*
*Nanyang Technological University*
<dawn.ng@nie.edu.sg>

Gloria Stillman
*University of Melbourne*
<g.stillman@unimelb.edu.au>



This paper reports mathematical and real world knowledge application by lower secondary students (aged 13-14 years) in a design-based interdisciplinary project with mathematics, science, and geography as anchor subjects implemented in Singapore[1]. Key findings from two mathematical tasks attempted by 10 case-study groups during the project reveal that whilst students displayed taught knowledge and skill, they lacked in-depth understanding of the use and purpose of scale drawings. There was also limited activation of real-world knowledge during mathematical decision making and little monitoring of accuracy and reasonableness of mathematical results.


The ability to solve problems is valued in the contemporary workplace by both workers and employers (Zevenbergen & Zevenbergen, 2009). Very often, such real-world problems "draw upon interdisciplinary knowledge" for solution and communication of results (English, 2009, p. 161) requiring individuals to synthesise both theoretical and practical understandings from their school learning (Sawyer, 2005). The young workers in the study by Zevenbergen and Zevenbergen often approached tasks in ways that were quite different from classroom practice using estimation and holistic thinking as well as problem solving and measuring and these authors foreshadow a need for a rethink of the practices of school mathematics if it is to be seen as developing numeracy for the workplace. In Singapore, interdisciplinary project work was introduced as an educational initiative in primary, secondary, and pre-university institutions in 2000 as part of Singapore's education master plan in order to prepare students for contemporary workplace demands in a knowledge-based economy (Quek et al., 2006). The application of mathematical knowledge during interdisciplinary learning invites a re-conceptualisation of *mathematical literacy*. Sawyer postulates that mathematical literacy consists of "the combination of knowing about mathematics and knowing when, where and how to apply this knowledge" (p. 649). In other words, students should be equipped with a repertoire of mathematical content knowledge and skills and exposed to a range of tasks where they can apply knowledge and skills in flexible and adaptive ways (Willoughby, 2000).

One way of promoting flexible and adaptive mathematical knowledge application for mathematical literacy is through contextualised mathematical problems within interdisciplinary settings. Research suggests that mathematical tasks embedded within real-life contexts that are meaningful for students can facilitate student access to the tasks and personalise their problem solving experiences in some instances (Stillman, 2000; Van den Heuvel-Panhuizen, 1997). This is because meaningful tasks are shaped by students' own relations with them (Beane, 1995; Busse, 2005). However, there can be challenges to producing *quality mathematical outcomes* during contextualised tasks with Venville, Rennie, and Wallace (2004) reporting that not all students view subject specific knowledge as useful in interdisciplinary tasks. Furthermore, others such as Wong (2001) have detected that students have difficulty in monitoring the appropriateness and accuracy of subject-based knowledge and skills. Some studies (e.g., Stillman, 2000; Verschaffel, Greer, & De Corte, 2000) have also found that students can ignore the contexts provided and not activate their

[1] This research was carried out when the first author was a PhD student with the University of Melbourne.


In R. Hunter, B. Bicknell, & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the Mathematics Education Research Group of Australasia* (Vol. 2). Palmerston North, NZ: MERGA.

real-world knowledge in mathematical decision making. The realisation that there are incongruities between what is being modelled by mathematics and the real situation sometimes overcomes this difficulty. Kordarki and Potari (1998), for example, found that in a design task involving 12 year old Greek students requiring the proposal of an area for leisure activities to the town mayor, a scale was able to be used to represent the real area of their proposed sports centres but often different scales were used for different parts of the centres making comparison of the sizes of the real areas impossible. However, extreme incompatibilities such as a kiosk being larger than the students' classroom led them to recognise the need for a common scale.

This paper investigates mathematical knowledge application by lower secondary Singaporean students during a design-based interdisciplinary project with mathematics, science, and geography as anchor subjects. The aim is to address the research question: What are the difficulties faced by students during the design-based real-world task when applying mathematical knowledge and skills?

## Design and Methodology

A multi-site, multi-case-based approach (Gillham, 2001) was used to obtain data on the nature of mathematical knowledge application during a researcher-designed mathematically-based interdisciplinary project. The project was implemented in 11 high-stream and five average-stream classes of three Singaporean government schools with students in grades seven and eight (ages 13-14). The students worked in groups of four for 14-15 weeks through project work classes (45-70 minutes weekly) facilitated by at least one mathematics, science, geography, or design and technology teacher to complete the project.

### *The Interdisciplinary Project*

An interdisciplinary project, "*Designing an Environmentally Friendly Building*", was designed by the first author for the purposes of this study based on the theme "*environmental conservation*". The aim of the project was to enhance students' environmental consciousness and explore methods of environmental conservation in the way Singaporeans live. The project required student-groups to decide on the type, name, purpose, location, and facilities of a building of their own design. Students also explored environmentally friendly features they could include before making physical scale models of their buildings from recycled materials based on scale drawings. The context of designing and building scale models of buildings was within the life experiences of students (e.g., doll-house building for girls, model-making for boys). Moreover, concepts and skills learnt during design and technology classes in the secondary curriculum could be useful during model making.

Two mathematical tasks in the project will be discussed here: (a) cost of furnishing and fitting out an area of the building and (b) scale drawings. In the first task, student-groups selected one area or room within their buildings for estimation of real furnishing costs and fit out. Such a task provided opportunities for students to engage in realistic cost evaluation and budgeting through research activities and discussion. The second task required appropriate hand-drawn scale drawings to be completed. These drawings were meant to guide the construction of physical scale models of the buildings. Here, students were expected to indicate two scales in their drawings, one for the model, and the other for the life size building. This was to help them become more aware of the practical uses of scale measurement, mensuration concepts and proportional reasoning. Students were to begin by estimating the dimensions (e.g., length, area, perimeter of common shapes) of their actual buildings in real-life.

### *Data Collection and Analysis Methods*

Only data from 10 student-groups ($n$ = 38) who consented to all data collection procedures were included for data analysis. There was an equal distribution of the groups from the high- and average-stream classes in the sample belonging to the three schools. Facilitating teachers confirmed that most of the selected groups were representative of the classes in terms of their quality of work and participation effort in the project.

Data sources included: (a) transcripts of video clips of student-groups at work during the two tasks, (b) post-task individual video-stimulated recall interviews, and (c) copies of student work collected immediately following the completion of the tasks. The first two sources of data were coded using the reformulated grounded theory approach of Corbin and Strauss (2008) to elicit patterns in the nature of mathematical knowledge application of the 10 student-groups. As the students spoke in Singapore English or Mandarin, elaborations and translations by the first author appear in brackets, {}, in transcripts illustrating results.

## Results

Findings reveal that whilst students displayed taught knowledge and skill, they lacked in-depth understanding of the purpose and use of scale drawings. There was also limited activation of real-world knowledge during mathematical decision making and little monitoring of accuracy and reasonableness of mathematical results. Details are discussed in the following.

### *Structured Steps used during Calculations*

The use of structured approaches during arithmetic calculations was evident in the cost of furnishing and fitting out task with 9 groups attempting the task during the allocated discussion time. Two of these nine produced only listings of items and costs with no calculations or total. However, application of taught procedures was evident in the workings of the other groups. Nine groups selected a rectangular shaped area for furnishing and fit out. Five of these groups used the area formula when calculating the flooring costs. Group 2 (see Figure 1) applied the concept of perimeter and related calculations to determine the cost of providing cloth as skirting along the sides of a fashion runway and stage located on the rooftop of their eco-friendly shopping centre. However, there was an undetected mistake in their calculations of the cloth for skirting as the group neglected to include the front of the runway (2 m in width) as part of their calculations.

### *Difficulty in Estimation*

Some students in the sample had difficulty with spatial visualisation which could have tempered unrealistic suggestions for estimations of lengths and widths of buildings and objects. Realistic dimensions of buildings, furnishings and fittings were needed for both the costing and the scale drawing tasks. Students efforts at estimating lengths and areas of buildings according to their selected shapes were not realistic and reasonable at times.

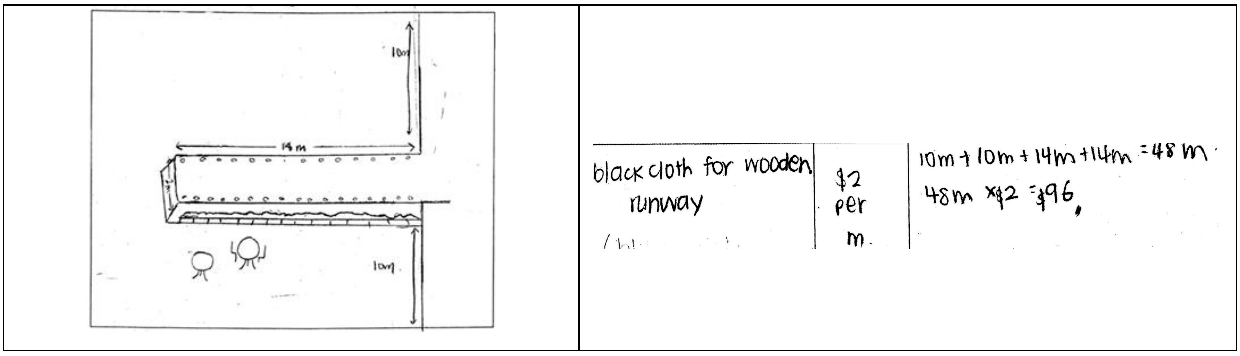



*Figure 1.* Use of perimeter concept and related algorithm by Group 2.

Tim and Matthew, for example, from Group 9 made suggestions such as 500km for an estimate for the length of their eco-friendly hotel without considering whether these were reasonable or feasible within the constraints of their actual chosen location of the building, the small island of Sentosa. However, Yoko, another group member alerted the group of possible space constraints of the location impacting on proposed dimensions of their building saying, "You have to see how big Sentosa is, right? Later no space to build, how? {We need to know the size of Sentosa Island [actually 4.63 $km^2$]. Our hotel should be able to fit into the small island of Sentosa, right?}" This brought a revision to 2 km for the length of the hotel but later 3km was suggested. Both boys were adamant 1.5 km was too small.

A similar situation occurred in Group 1 where Kath insisted on having 25 $m^2$ as the floor area of a school hall without considering whether this could accommodate the total student population (usually 1000) during assembly. As seen below, Yang's alerting his group to the unrealistic dimensions of the school went unheeded.

Kath: [Loudly] Five! Five floors!

Yang: [Records Kath's suggestion.]

Kath: Then the width is 5…

Chi: [Interrupting Kath's explanations.] Kilometres or metres? Divided by 5 or by 10? Divide by 10 lah! {Is this in kilometres or metres? Is the building 5 or 10 stories high? How about 10 stories?}

Kath: 5 stories mah! Then the wide is 5 metres…{There are five classes, so 5 stories in the building. The width (height) of the building should be 5…we can use class size to estimate the width of the building…5 class sizes means that the building is 5 metres wide.}

Chi: In square…{Then the area of the building is in square metres?}

Yang: [Gesturing with his hands.]…yeah…then the area is a square….{The area of the school is 5 metres by 5 metres, in the shape of a square. This area is too small. There is not enough space for students in the canteen.}

Hasyim: [To Yang.] Write the width as 5 metres first lah…

## *Activation of Real-World Knowledge*

There was limited activation of real-world knowledge by the 10 student-groups (see Table 1), despite the presentation of the project within a real-world scenario of designing an eco-friendly building. Within the theme of the interdisciplinary project, such knowledge refers to the incorporation of eco-friendly features in building design and furnishings as well as displays of real-world considerations in mathematical decision making.

Table 1
*Frequency of Aspects Displayed by the Groups During Discussions on the Two Tasks*

| Aspects Displayed | Number of Instances by Group | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | High-Educational Stream | | | | | Average-Educational Stream | | | | |
| | 2 | 3 | 5 | 6 | 9 | 1 | 4 | 7 | 8 | 10 |
| | Cost of Furnishing Task | | | | | | | | | |
| Eco Friendly features used | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Real world considerations | 2 | 1 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| Checking of outcome | 5 | 1 | 0 | 5 | 0 | 1 | 4 | 0 | 0 | 0 |
| Unchecked outcome | 3 | 4 | 1 | 1 | 3 | 1 | 1 | 1 | 5 | 3 |
| Reasonable outcome | 3 | 1 | 0 | 2 | 0 | 0 | 1 | 0 | 0 | 0 |
| | Scale Drawing Task | | | | | | | | | |
| Real world considerations | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Checking of outcome | 3 | 3 | 3 | 0 | 0 | 1 | 2 | 0 | 0 | 0 |
| Unchecked outcome | 0 | 1 | 2 | 0 | 0 | 1 | 0 | 4 | 1 | 1 |
| Reasonable outcome | 3 | 3 | 4 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |

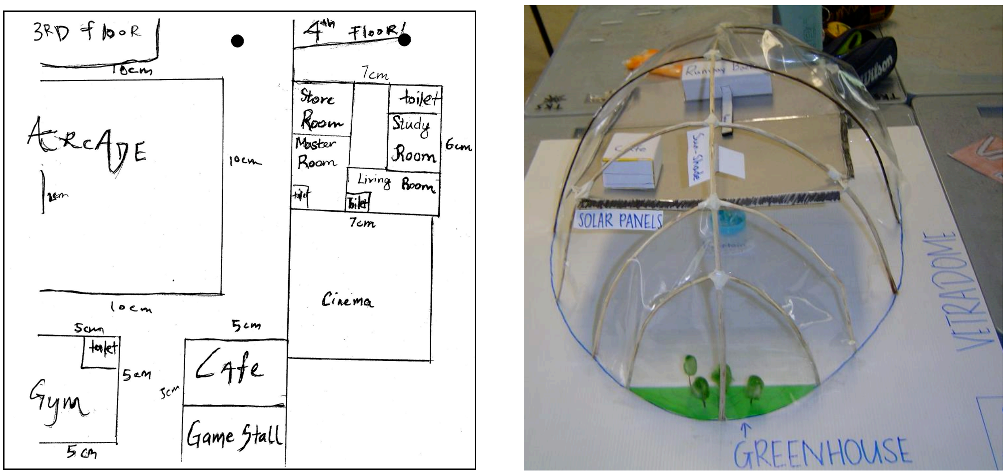
Groups 2, 3, 6 and 9 from the high-stream were the only groups to apply real-world knowledge during mathematical tasks in the project. Activation of real-world knowledge in mathematical decision making by Group 2, for example, was evident when the group factored in their life experiences, namely, comparing with real-world objects, when evaluating the feasibility of their proposed estimations of the dimensions of their eco-friendly shopping centre. Sani and Fanny based their estimation on the size of an existing shopping complex in Singapore (i.e., Ngee Ann City) using proportional reasoning. To further convince group member, Choon, of their point, the girls sought the help of their teacher. The teacher also concluded that one kilometre, their suggested length of the shopping centre, was a short distance to walk saying, "One kilometre is very short…1 kilometre is two rounds on the [school] field…It's very small only" and therefore would be a reasonable estimate. However, Choon still deliberated on the matter for another five minutes before giving in to the persuasions of his peers. His primary concern was with the availability of space for housing their shopping centre as land is scarce in Singapore. Later the length of the centre was revised to 80m when the students encountered practical constraints of materials used in building their model. Choon's consideration of space was in keeping with the nature of interdisciplinary projects. Such projects draw in disciplinary knowledge and skills to address real-world issues. In bringing up the issue of available space when considering building dimensions, Choon had successfully integrated his subject-specific knowledge from geography with mathematics.

### *Monitoring of Accuracy and Reasonableness of Mathematical Decisions*

During the study, there was more checking by high-stream groups rather than the average-stream groups as to whether their knowledge application was reasonable or realistic (see Table 1). Groups which did not check for accuracy and reasonableness of mathematics applied, mainly had their decision making processes dominated by particular members of the group. Such students made quick estimations of the cost of furnishing items and dimensions of their building in real-life without much discussion and evaluation with the others. The checking that was undertaken did not always result in reasonable outcomes in terms of an appropriate decision being made, a realistic dimension chosen or an accurate mathematical result. On the other hand, as seen in Table 1, not all unchecked outcomes were unreasonable.

## *Awareness of Purpose of Scale*

It was assumed that the student-groups would recognise that the area represented in their drawings and the real-life area of their building compounds had to be mathematically represented through use of scale measurements. A one-to-one correspondence between the dimensions in the drawing and those in real-life was impossible. Instructions for the use of a scale to represent the various dimensions of the real-life building on drawings were given by facilitating teachers. However, only four out of 10 groups managed to complete proper scale drawings for the project. Three of these groups were from the high-educational stream. The other six groups made by-hand sketches or measured drawings (Figure 2a) and in one case an MsPaint drawing, showing no indications of their awareness of the purpose of using scales and in most cases giving no indication of dimensions.




(a) (b)

*Figure 2.* Responses to model building task: (a) measured diagram (b) shopping centre with plastic dome.

Data from student-interviews revealed that some students were not aware that two scales have to be selected to represent the dimensions of the physical scale model and the real-life building in the drawings. For instance, a length of 100 m can be represented by 1cm on the drawing and 10cm on the model. This means two scales (i.e., 1cm : 100m and 1cm : 10m) could be used in the same drawing. However, only two groups (Groups 7 and 2) showed two scales. Group 7 indicated two scales on their plan of an eco-friendly mansion as 1cm : 5m for the actual house and 1cm : 15cm on the scale model. This second scale would have produced a large but feasible model with the longest dimension being 1.3m. When it came to model construction however, like many of the groups in the sample and nearly all the groups in the participating classes, this group did not refer to the recorded dimensions for the model as they had difficulty resizing recycled materials, namely tissue and chocolate boxes, to the required dimensions. Unfortunately, for Group 2 their scales for the model did not realistically represent the dimensions of their building. They used the scales 1: 400 and 1: 200 to represent the real-life building and the model, respectively. For this group, 1 cm on the model actually represented 2m on the building. This was neither realistic nor reasonable in a real-world context because it meant that the group would have to construct a 4-metre long model to represent their building which was 80m in length. As revealed later in interview, when building the actual model its dimensions were restricted to much less than 4m by the lengths of sticks used to support a plastic dome over the shopping centre (see Figure 2b) which Choon had not realised was a constraint when making the scale drawings. The other two groups who made proper scale drawings (Groups 3 and 5) stated only one scale representing the dimensions of the model. Hence, it was questionable whether many students from the sample were aware of the relationship between the two scales or the need for complying with a scale in model construction.

### *Use of Different Scales for the Same Building*

One of the four groups who made proper scale drawings, Group 3, used different scales for various orthogonal projections of the top, side, and front views of their building. Initially, the group did not actively consider the impact of this decision on comparisons of real-life measurements and proportionality of the different parts of the building as well as the construction of the model. Alice, the dominant member of the group, decided that all group members were free to choose their own scale as they drew the various projections of the building they were assigned. Much later when the drawings were partially completed, Alice encountered difficulty matching her view with that of others as common features became too small to draw accurately on her diagram. She thus asked the group to adjust their scales which were twice hers in order to produce a coherent image of the building through their drawings.

## Discussion and Conclusion

Research has suggested that contextualised tasks enhance students' engagement (Van den Heuvel-Panhuizen, 1997) with the tasks. However, despite heightened interest in the activities within their embedded contexts, Abrantes (1993) found weaknesses in the "intellectual quality" (p. 112) of the group discussions, leading to poor-quality mathematical outcomes. Indeed, this study found that although some students showed strengths in algorithmic manipulations, their application of mathematics in the project was somewhat limited by (a) poor understanding of the purpose and use of scale, (b) difficulty in estimation based on spatial visualisation governed by real-world constraints, (c) lack of activation of real-world knowledge in mathematical decision making, and (d) minimal displays of monitoring behaviours to check for accuracy and reasonableness of mathematics applied.

Such findings appear to be in line with those from existing studies (e.g., Kordaki & Potari, 1998; Stillman, 2000; Venville, Rennie, & Wallace, 2004) and perhaps point towards the need for suitable intervention by project facilitators targeting monitoring behaviours and critical analysis of real-world knowledge during interdisciplinary projects. Within the specifics of the context of this project, it would also appear that mathematics teachers need to develop students' understanding of the purpose and use of scale measurements as well as mathematical application in contextualised tasks.

Teachers have to be aware that relevant and conscious knowledge application of mathematical content and skills within interdisciplinary tasks, espoused to be appropriate for developing numeracies for the contemporary workforce (Zevenbergen & Zevenbergen, 2009), is not automatic. There appears to be a gap between school-based learning and what students early in their experience of interdisciplinary projects can flexibly apply in such projects to achieve the desired integration of disciplinary knowledge and skills to address real-world issues. In addition, fruitful mathematical returns from real-world tasks which activate group collaborative mathematical knowledge application processes depends on the nature of monitoring behaviours of key players in the group such that desirable mathematical outcomes are produced.

# References


Abrantes, P. (1993). Learning activities involving mathematics in real-life situations. In T. Breiteig, I. Huntley & G. Kaiser-Messmer (Eds.), *Teaching and learning mathematics in context* (pp. 103-114). Chichester, UK: Ellis Horwood.

Beane, J. A. (1995). Curriculum integration and the disciplines of knowledge. *Phi Delta Kappan, 76*(8), 616-622.

Busse, A. (2005). Individual ways of dealing with the context of realistic tasks: First steps towards a typology. *Zentralblatt für Didaktik der Mathematik*, 37(5), 354-360.

Corbin, J. & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory* (3rd ed.). Thousand Oaks, CA: Sage.

English, L. (2009). Promoting interdisciplinarity through mathematical modelling. *ZDM—The International Journal on Mathematics Education, 41*(1-2), 161-181.

Gillham, B. (2001). *Case study research methods*. London: Continuum.

Gravemeijer, K. P. E. (1994). *Developing realistic mathematics education*. Utrecht, The Netherlands: Freudenthal Institute.

Kodarki, M., & Potari, D. (1998). Children's approaches to area measurement through different contexts. *Journal of Mathematical Behavior, 17*(3), 303-316.

Quek, C., Divaharan, S., Liu, W., Peer, J., Williams, M., Wong, A., et al. (2006). *Engaging in project work*. Singapore: McGaw Hill.

Sawyer, A. (2005). Education for early mathematical literacy: More than maths know-how. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonough, R. Pierce, et al. (Eds.), *Building connections, research, theory and practice: Proceedings of the 28th Mathematics Education Research Group of Australasia Conference, Melbourne,* (Vol. 2, pp. 649-655). Sydney: MERGA.

Stillman, G. (2000). Impact of prior knowledge of task context on approaches to applications tasks. *Journal of Mathematical Behavior, 19*(3), 333-361.

Van den Heuvel-Panhuizen, M. (1997, September). *Realistic problem solving and assessment.* Paper presented at the Mathematics Teaching Conference, University of Edinburgh, Scotland.

Venville, G., Rennie, L., & Wallace, J. (2004). Decision making and sources of knowledge: How students tackle integrated tasks in science, technology and mathematics. *Research in Science Education, 34*(2), 115-135.

Verschaffel, L., Greer, B., & De Corte, E. (2000). *Making sense of word problems*. Lisse, The Netherlands: Swets & Zeitlinger.

Willoughby, S. S. (2000). Perspectives on mathematics education. In M. J. Burke & F. R. Curcio (Eds.), *Learning mathematics for a new century* (pp. 1-15). Reston, VA: NCTM.

Wong, H. M. (2001). *Project work in primary school: Effects on learning and teaching.* Unpublished master's thesis, Nanyang Technological University, National Institute of Education, Singapore.

Zevenbergen, R., & Zevenbergen, K. (2009). The numeracies of boatbuilding: New numeracies shaped by workplace technologies. *International Journal of Science and Mathematics Education, 7*(1), 183-206.